

Lecture 5. Coulomb blockade and sequential transport in quantum dots

Single-particle states. Let the single-electron Hamiltonian,

$$\hat{H}_1 = \sum_{\sigma} \int d^d \mathbf{r} \hat{\psi}_{\sigma}^{\dagger}(\mathbf{r}) \left[-\frac{\nabla^2}{2m} + V(\mathbf{r}) \right] \hat{\psi}_{\sigma}(\mathbf{r}) = \sum_{n,\sigma} \epsilon_n \hat{c}_{n\sigma}^{\dagger} \hat{c}_{n\sigma}, \quad \hat{\psi}_{\sigma}(\mathbf{r}) = \sum_n \phi_n(\mathbf{r}) \hat{c}_{n\sigma}, \quad (5.1)$$

include the confinement potential $V(\mathbf{r})$, which determines the dot volume L^d by the condition $V(\mathbf{r}) < \epsilon_F$. If electron motion in the dot is chaotic, each $\phi_n(\mathbf{r})$ is a random superposition of all plane waves in the energy window of the width $E_T \sim v_F/L$ (Thouless energy) around ϵ_F , with coefficients $\zeta_{n\mathbf{p}}$ which are complex Gaussian random variables with no correlations except for $\zeta_{n\mathbf{p}} = \zeta_{n,-\mathbf{p}}^*$ if there is no magnetic field:

$$\phi_n(\mathbf{r}) = \sum_{\mathbf{p}: |p^2/2m - \epsilon_F| < E_T/2} \zeta_{n\mathbf{p}} \frac{e^{i\mathbf{p}\mathbf{r}}}{L^{d/2}}, \quad \overline{\zeta_{n\mathbf{p}}^* \zeta_{n'\mathbf{p}'}} = \overline{\zeta_{n\mathbf{p}} \zeta_{n',-\mathbf{p}'}} = \mathcal{A} \delta_{nn'} \delta_{\mathbf{p}\mathbf{p}'}. \quad (5.2)$$

The constant \mathcal{A} is determined from the normalization:

$$1 = \int \overline{\phi_n^2(\mathbf{r})} d^d \mathbf{r} = \mathcal{A} \int \frac{L^d d^d \mathbf{p}}{(2\pi)^d} E_T \delta(p^2/2m - \epsilon_F) = \mathcal{A} \frac{E_T}{\delta_1} \equiv \mathcal{A}g. \quad (5.3)$$

Here $1/\delta_1$ is the average density of states (per spin) in the dot, δ_1 is the typical spacing between random energy levels ϵ_n . This description is valid when $|\epsilon_n - \epsilon_F| \ll E_T$, so we need $\delta_1 \ll E_T$. In the presence of disorder with the mean free path $\ell \ll L$, the electron motion in the dot is diffusive with the diffusion coefficient $D \sim v_F \ell$. Then $E_T \sim D/L^2$; indeed, for $|\epsilon_n - \epsilon_{n'}| \gg D/L^2$ the wave function correlations are those of an infinite d -dimensional diffusive medium. The dimensionless conductance of the dot $g \sim \nu_0 D L^{d-2} = E_T/\delta_1 \gg 1$.

Let the dot be connected to two massive leads, left and right (L, R), by weak tunneling contacts. Thus, in addition to the Hamiltonian \hat{H}_1 , we have the Hamiltonian of the leads, and the tunnel Hamiltonian, which describes the coupling between the dot and the leads:

$$\hat{H}_L + \hat{H}_R + \hat{H}_{\text{tun},L} + \hat{H}_{\text{tun},R} = \sum_{\alpha=L,R} \left[\sum_{k,\sigma} \xi_{\alpha,k} \hat{c}_{\alpha k\sigma}^{\dagger} \hat{c}_{n\sigma} + \sum_{k,n,\sigma} (t_{\alpha kn} \hat{c}_{\alpha k\sigma}^{\dagger} \hat{c}_{n\sigma} + \text{h.c.}) \right]. \quad (5.4)$$

Electrons in each lead are assumed to be in equilibrium, characterized by some chemical potential and temperature. If, in addition, the leads are in equilibrium between themselves, the chemical potentials in the leads are equal, so all single-particle energies can be counted from this common Fermi level. If some voltages V_R, V_L are applied to the leads, the distribution function in the leads is $f_{L,R}(\xi) = f_0(\xi - eV_{L,R})$, where $f_0(\xi) = 1/(1 + e^{\xi/T})$. If $V_{L,R} = 0$, but the distribution function in the dot is shifted by a voltage $V \gg \delta/e$, $f_n = f_0(\epsilon_n - eV)$, the current from the dot to the α th lead is

$$I_{\alpha} = 2e \sum_{n,k} |t_{\alpha kn}|^2 2\pi \delta(\epsilon_n - \xi_{\alpha k}) [f_0(\epsilon_n - eV) - f_0(\xi_{\alpha k})] = 2e \sum_{0 < \epsilon_n < eV} \gamma_{n\alpha} = \frac{2e^2}{2\pi} \frac{2\pi \bar{\gamma}_{\alpha}}{\delta_1} V, \quad (5.5)$$

where $\gamma_{n\alpha}$ is the the escape rate from the level n to the lead α as determined by Fermi Golden Rule, whose average determines the dimensionless conductance g_{α} of the contact

$$\gamma_{n\alpha} = \sum_k |t_{\alpha kn}|^2 2\pi \delta(\epsilon_n - \xi_{\alpha k}), \quad g_{\alpha} = 2\pi \bar{\gamma}_{\alpha} / \delta_1. \quad (5.6)$$

Porter-Thomas distribution. As $\gamma_{n\alpha}$ are random quantities, we have to determine their statistics. Let us assume that the tunneling to each electrode occurs in the vicinity of some contact point \mathbf{r}_α , so that

$$t_{\alpha kn} = t_\alpha \psi_{k\alpha}^*(\mathbf{r}_\alpha) \phi_n(\mathbf{r}_\alpha), \quad (5.7)$$

where $\psi_{k\alpha}(\mathbf{r})$ is the wave function in the lead. As $\phi_n(\mathbf{r})$ is a linear combination of Gaussian random variables, it is also Gaussian with pair correlator

$$\overline{\phi_n(\mathbf{r}) \phi_{n'}(\mathbf{r}')^*} = \delta_{nn'} \delta_1 \int \frac{d^d \mathbf{p}}{(2\pi)^d} \delta(p^2/2m - \epsilon_F) e^{i\mathbf{p}(\mathbf{r}-\mathbf{r}')} = \frac{\delta_{nn'}}{L^d} \mathcal{K}_d(p_F |\mathbf{r} - \mathbf{r}'|). \quad (5.8)$$

where $\mathcal{K}_2(x) = J_0(x)$, $\mathcal{K}_3(x) = (\sin x)/x$. Thus, $\overline{t_{\alpha kn}^* t_{\alpha' k' n'}}$ $\propto \delta_{\alpha\alpha'} \delta_{nn'}$, so that $\gamma_{n\alpha}$ are uncorrelated for different n or α , but they are not Gaussian. Writing

$$\gamma_{n\alpha} = |\phi_n(\mathbf{r}_\alpha)|^2 \sum_k |t_\alpha \psi_{k\alpha}^*(\mathbf{r}_\alpha)|^2 2\pi \delta(\epsilon_n - \xi_{\alpha k}) = |\phi_n(\mathbf{r}_\alpha)|^2 L^d \bar{\gamma}_\alpha, \quad (5.9)$$

where we assume that the local density of state in the lead does not depend on energy, we see that each $\gamma_{n\alpha}$ is the square of a Gaussian random variable, so it is distributed according to the Porter-Thomas distribution:

$$\mathcal{P}(\gamma) = \theta(\gamma) \frac{e^{-\gamma/(2\bar{\gamma})}}{\sqrt{2\pi\bar{\gamma}\gamma}}. \quad (5.10)$$

Transport through a non-interacting dot. Let us assume the electronic levels in the dot to be sufficiently broadened, then the distribution function in the dot can be found from the kinetic equation:

$$\frac{\partial f_n}{\partial t} = \sum_{\alpha, k} |t_{\alpha kn}|^2 2\pi \delta(\epsilon_n - \xi_{\alpha k}) [f_\alpha(\xi_{\alpha k}) - f_n], \quad (5.11)$$

while the distribution functions in the leads, $f_\alpha(\xi)$, are assumed to be fixed. The stationary solution is given by

$$f_n = \frac{\gamma_{nL}}{\gamma_{nL} + \gamma_{nR}} f_L(\epsilon_n) + \frac{\gamma_{nR}}{\gamma_{nL} + \gamma_{nR}} f_R(\epsilon_n). \quad (5.12)$$

In the stationary situation, due to the current conservation, the current from the left lead to the right lead is equal to that from the left lead to the dot:

$$I_{L \rightarrow R} = 2 \sum_{n, k} |t_{Lkn}|^2 2\pi \delta(\epsilon_n - \xi_{Lk}) [f_L(\xi_{Lk}) - f_n] = 2 \sum_n \frac{\gamma_{nL} \gamma_{nR}}{\gamma_{nL} + \gamma_{nR}} [f_L(\epsilon_n) - f_R(\epsilon_n)] \quad (5.13)$$

If the voltage or temperature are large compared to δ_1 , the sum over n performs the effective averaging [we denote $\tau_\alpha \equiv 1/\bar{\gamma}_\alpha$]:

$$\begin{aligned} \frac{\gamma_L \gamma_R}{\gamma_L + \gamma_R} &= \int_0^\infty \frac{\gamma_L e^{-\gamma_L t} \gamma_R e^{-\gamma_R t}}{\gamma_L e^{-\gamma_L t} + \gamma_R e^{-\gamma_R t}} dt = \int_0^\infty \frac{\sqrt{\tau_L \tau_R} dt}{(2t + \tau_L)^{3/2} (2t + \tau_R)^{3/2}} = \\ &= \frac{2\sqrt{\tau_L \tau_R}}{(\tau_L - \tau_R)^2} \int_{\frac{\tau_L + \tau_R}{|\tau_L - \tau_R|}}^\infty \frac{du}{(u^2 - 1)^{3/2}} = \frac{1}{(\sqrt{\tau_L} + \sqrt{\tau_R})^2}. \end{aligned} \quad (5.14)$$

Electron-electron interaction. The average matrix elements of the pair interaction potential, $U(\mathbf{r}, \mathbf{r}')$ (having in mind the screened Coulomb potential, we do not assume the translational invariance, since the screening depends on the dot shape), are given by

$$\begin{aligned} \overline{U_{n_1 n_2 n_3 n_4}} &= \int U(\mathbf{r}, \mathbf{r}') \overline{\phi_{n_1}^*(\mathbf{r}) \phi_{n_2}^*(\mathbf{r}') \phi_{n_3}(\mathbf{r}') \phi_{n_4}(\mathbf{r})} d^d \mathbf{r} d^d \mathbf{r}' = \\ &= \int \frac{d^d \mathbf{r}}{L^d} \frac{d^d \mathbf{r}'}{L^d} U(\mathbf{r}, \mathbf{r}') \left[\delta_{n_1 n_4} \delta_{n_2 n_3} + (\delta_{n_1 n_3} \delta_{n_2 n_4} + \delta_{n_1 n_2} \delta_{n_3 n_4}) \mathcal{K}_d^2(p_F |\mathbf{r} - \mathbf{r}'|) \right] = \\ &= (2E_C + J_S/2) \delta_{n_1 n_4} \delta_{n_2 n_3} + J_S \delta_{n_1 n_3} \delta_{n_2 n_4} + J_C \delta_{n_1 n_2} \delta_{n_3 n_4}, \end{aligned} \quad (5.15)$$

where $\mathcal{K}_2(x) = J_0(x)$, $\mathcal{K}_3(x) = (\sin x)/x$, and we used the pair wave function correlator:

$$\overline{\phi_n(\mathbf{r}) \phi_{n'}(\mathbf{r}')} = \delta_{nn'} \delta_1 \int \frac{d^d \mathbf{p}}{(2\pi)^d} \delta(p^2/2m - \epsilon_F) e^{i\mathbf{p}(\mathbf{r}-\mathbf{r}')} = \frac{\delta_{nn'}}{L^d} \mathcal{K}_d(p_F |\mathbf{r} - \mathbf{r}'|). \quad (5.16)$$

As a result, we obtain the universal Hamiltonian,

$$\overline{\hat{H}} = E_C \hat{N}^2 - J_S \hat{\mathbf{S}}^2 + J_C \hat{F}^\dagger \hat{F}, \quad (5.17)$$

$$\hat{N} = \sum_{n,\sigma} \hat{c}_{n\sigma}^\dagger \hat{c}_{n\sigma}, \quad \hat{S}_i = \sum_{n,\sigma_1,\sigma_2} \hat{c}_{n\sigma_1}^\dagger \frac{\sigma_{\sigma_1\sigma_2}^i}{2} \hat{c}_{n\sigma_2}, \quad \hat{F} = \sum_n \hat{c}_{n\uparrow} \hat{c}_{n\downarrow}. \quad (5.18)$$

To write down the second term, we have rearranged the spin indices according to

$$2\delta_{\sigma_1\sigma_4} \delta_{\sigma_2\sigma_3} = \delta_{\sigma_1\sigma_2} \delta_{\sigma_3\sigma_4} + \sum_{i=x,y,z} \sigma_{\sigma_1\sigma_2}^i \sigma_{\sigma_3\sigma_4}^i. \quad (5.19)$$

The term $E_C \hat{N}^2$ can be understood from the classical electrostatics. The potential φ on a metallic island with a charge Q on it, in the presence of several metallic gates with fixed potentials φ_i is $\varphi = \varphi_i + Q_i/C_i$, where C_i is the capacitance between the island and the i th gate, and Q_i is the charge on each capacitor, with $\sum_i Q_i = Q$. The energy is

$$\mathcal{E}(Q) = \int_0^Q \varphi(Q') dQ' = \frac{Q^2}{2C} + \sum_i C_i \varphi_i \frac{Q}{C}, \quad C = \sum_i C_i. \quad (5.20)$$

Thus, the screened Coulomb potential can be written in the form:

$$U(\mathbf{r}, \mathbf{r}') = \frac{e^2}{C} + \int \frac{d^d \mathbf{q}}{(2\pi)^d} \frac{U_0(\mathbf{q}) e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}')}}{1 + U_0(\mathbf{q})(2/\delta_1 L^d)} + U'(\mathbf{r}) + U'(\mathbf{r}'). \quad (5.21)$$

The first term appears because the constant part of the potential cannot be screened at a constant number of electrons on the dot. The second can be approximated by $(\delta_1 L^d/2) \delta(\mathbf{r} - \mathbf{r}')$ at distances larger than the screening radius a_s . The last two terms depend on how the extra charge is redistributed over the dot, may be concentrated near the boundary (an isolated metallic sphere) or determined by the gate geometry.

J_S and J_C can be estimated by noting that (i) the infinite-range part of the interaction does not contribute due to orthogonality of the wave functions [which holds only approximately in our Gaussian approximation, by a factor $1/(p_F L)^{d-1} = 1/g$]; (ii) $p_F a_s > 1$, so the δ -function approximation for the short-range part overestimates J_S and J_C ; (iii) positive J_C is further renormalized to zero by coupling to high-energy states. As a result, typically $J_S, J_C \ll \delta_1$, while $E_C \gg \delta_1$. Usually, $E_C < E_T$.

When the dot is connected to the leads, the number of electrons N should be determined self-consistently. Let the leads be in equilibrium at $T = 0$. The energy of the ground state of the dot with a given number of electrons, $\mathcal{E}_N = E_C N^2 + N e \varphi_g$, determines the chemical potential of the dot,

$$\mu(N) = \frac{\partial \mathcal{E}_N}{\partial N} = 2NE_C + e\varphi_g, \quad (5.22)$$

In equilibrium, N should be such that $\mu(N) = \epsilon_F$, given by the leads. The solution, $N_g = (\epsilon_F - e\varphi_g)/(2E_C)$ is not necessarily integer. For N close to N_g , we approximate

$$\mathcal{E}_N \approx \mathcal{E}_{N_g} + \left. \frac{\partial^2 \mathcal{E}_N}{\partial N^2} \right|_{N_g} \frac{(N - N_g)^2}{2} = \mathcal{E}_{N_g} + E_C (N - N_g)^2. \quad (5.23)$$

It is crucial that in Eq. (5.23), N is a positive integer, while N_g is a continuous variable, which can be controlled by an external gate.

Let us put ΔN excess electrons on the dot. The decay rate of this many-body state due to electron escape into the leads is given by the sum of the rates for all electrons which are allowed to escape. As upon transferring one electron the energy $2E_C \Delta N$ is released, (in other words, the chemical potential of the dot is $2E_C \Delta N$), the number of such electrons is $2E_C \Delta N / (\delta_1/2)$, so the decay rate is $4E_C \Delta N \gamma / \delta_1$. The energy of the state is $E_C (\Delta N)^2$. Thus, the Coulomb blockade is strong when $\gamma / \delta_1 \gg 1$, otherwise it is washed out by quantum fluctuations.

Rate equations. When $T \gg E_C$, the Coulomb blockade does not play a role, and the transport through the dot can be described by the usual non-interacting Boltzmann equation. Let us work near a degeneracy point, that is, φ_g is such that $\mathcal{E}_{N+1} - \mathcal{E}_N = \Delta$, $|\Delta| \ll E_C$. Then only two values of the charge are effective, N and $N + 1$.

First, consider the limit $\delta_1 \ll T \ll E_C$. Let p_N and p_{N+1} be the probabilities for the charge on the dot to be N and $N + 1$, respectively ($p_N + p_{N+1} = 1$). Then we can define $f_{n|N}$ and $f_{n|N+1}$ as conditional occupation probabilities of the level n in the two cases.

$$\begin{aligned} \frac{dp_N}{dt} = & -2p_N \sum_{\alpha, k, n} |t_{\alpha kn}|^2 2\pi \delta(\Delta + \epsilon_n - \xi_{\alpha k}) f_{\alpha}(\xi_{\alpha k}) [1 - f_{n|N}] + \\ & + 2p_{N+1} \sum_{\alpha, k, n} |t_{\alpha kn}|^2 2\pi \delta(\Delta + \epsilon_n - \xi_{\alpha k}) [1 - f_{\alpha}(\xi_{\alpha k})] f_{n|N+1}. \end{aligned} \quad (5.24)$$

Let us assume that the distribution function on the dot is maintained in the Fermi-Dirac form by electron-electron collisions. Then $f_{n|N}$ and $f_{n|N+1}$ are both Fermi-Dirac, but with slightly different chemical potentials, to give the different number of particles. However, this difference is $\sim \delta_1 \ll T$, so we neglect it. Then, in the stationary regime we have

$$p_N \sum_{\alpha} \bar{\gamma}_{\alpha} \int d\epsilon f_0(\epsilon + \Delta - eV_{\alpha}) [1 - f_0(\epsilon)] = p_{N+1} \sum_{\alpha} \bar{\gamma}_{\alpha} \int d\epsilon [1 - f_0(\epsilon + \Delta - eV_{\alpha})] f_0(\epsilon), \quad (5.25)$$

where the averaging over many levels is justified by $T \gg \delta_1$. This gives

$$p_N = \frac{\sum_{\alpha} \bar{\gamma}_{\alpha} F(\Delta - eV_{\alpha})}{\sum_{\alpha} \bar{\gamma}_{\alpha} [F(\Delta - eV_{\alpha}) + F(eV_{\alpha} - \Delta)]}, \quad (5.26)$$

where

$$F(\omega) = \int f_0(\epsilon - \omega) [1 - f_0(\epsilon)] d\epsilon = \int_0^\infty \frac{T e^{\omega/T} dx}{(x+1)(x+e^{\omega/T})} = \frac{\omega}{1 - e^{-\omega/T}}. \quad (5.27)$$

In the stationary situation, due to the current conservation, the current from the left lead to the right lead is equal to that from the dot to the right lead:

$$\begin{aligned} I_{L \rightarrow R} &= -2ep_N \sum_{k,n} |t_{Rkn}|^2 2\pi\delta(\Delta + \epsilon_n - \xi_{Rk}) f_R(\xi_{Rk}) [1 - f_{n|N}] + \\ &\quad + 2ep_{N+1} \sum_{k,n} |t_{Rkn}|^2 2\pi\delta(\Delta + \epsilon_n - \xi_{Rk}) [1 - f_R(\xi_{Rk})] f_{n|N+1} = \\ &= \frac{2e\bar{\gamma}_R}{\delta} [-p_N F(eV_R - \Delta) + p_{N+1} F(\Delta - eV_R)] = \\ &= \frac{2e\bar{\gamma}_R\bar{\gamma}_L}{\delta} \times \\ &\quad \times \frac{F(eV_L - \Delta) F(\Delta - eV_R) - F(eV_R - \Delta) F(\Delta - eV_L)}{\bar{\gamma}_R [F(eV_R - \Delta) + F(\Delta - eV_R)] + \bar{\gamma}_L [F(eV_L - \Delta) + F(\Delta - eV_L)]} \approx \\ &\approx \frac{2e^2\bar{\gamma}_R\bar{\gamma}_L}{\delta(\bar{\gamma}_R + \bar{\gamma}_L)} \frac{F'(\Delta) F(-\Delta) + F'(-\Delta) F(\Delta)}{F(-\Delta) + F(\Delta)} (V_L - V_R) = \\ &= \frac{2e^2}{\delta} \frac{\bar{\gamma}_R\bar{\gamma}_L}{\bar{\gamma}_R + \bar{\gamma}_L} \frac{\Delta/T}{2 \text{sh}(\Delta/T)} (V_L - V_R). \end{aligned} \quad (5.28)$$

Note that here the combination $\bar{\gamma}_R\bar{\gamma}_L/(\bar{\gamma}_R + \bar{\gamma}_L)$ of average γ 's enters, while in Eq. (5.13) it is the whole combination $\gamma_{nL}\gamma_{nR}/(\gamma_{nL} + \gamma_{nR})$ which should be averaged.

In the limit $T \ll \delta_1 \ll E_C$, only the ground states of the dot with N and $N+1$ electrons count. Let N be odd, and the half-filled level be $n = n_F$. Then three dot states are important: $|1_{n_F,\uparrow}, 0_{n_F,\downarrow}\rangle$, $|0_{n_F,\uparrow}, 1_{n_F,\downarrow}\rangle$, $|1_{n_F,\uparrow}, 1_{n_F,\downarrow}\rangle$. Let us denote their probabilities by $p_\uparrow, p_\downarrow, p_{\uparrow\downarrow}$. The case of even N will give the same due to electron-hole symmetry. The energy ϵ_{n_F} simply renormalizes Δ , so we set $\epsilon_{n_F} = 0$. The rate equations are

$$\frac{dp_\sigma}{dt} = -p_\sigma \sum_\alpha \gamma_\alpha f_\alpha(\Delta) + p_{\uparrow\downarrow} \sum_\alpha \gamma_\alpha [1 - f_\alpha(\Delta)], \quad (5.29)$$

$$\frac{dp_{\uparrow\downarrow}}{dt} = -2p_{\uparrow\downarrow} \sum_\alpha \gamma_\alpha [1 - f_\alpha(\Delta)] + (p_\uparrow + p_\downarrow) \sum_\alpha \gamma_\alpha f_\alpha(\Delta). \quad (5.30)$$

Here $\gamma_\alpha \equiv \gamma_{n_F\alpha}$. Note that δ_1 does not enter at all; however, processes involving excited states of the dot will have rates $e^{-\delta_1/T}$. The stationary solution of these equations is

$$p_\uparrow = p_\downarrow = \frac{1 - p_{\uparrow\downarrow}}{2} = \frac{\sum_\alpha \gamma_\alpha f_\alpha(\Delta)}{\sum_\alpha \gamma_\alpha [2 - f_\alpha(\Delta)]}. \quad (5.31)$$

The current

$$\begin{aligned} I_{L \rightarrow R} &= -2p_{\uparrow\downarrow}\gamma_L [1 - f_L(\Delta)] + (p_\uparrow + p_\downarrow)\gamma_L f_L(\Delta) = \\ &= 2p_{\uparrow\downarrow}\gamma_R [1 - f_R(\Delta)] - (p_\uparrow + p_\downarrow)\gamma_R f_R(\Delta) = \\ &= \frac{2e\gamma_L\gamma_R [f_L(\Delta) - f_R(\Delta)]}{\gamma_L [2 - f_L(\Delta)] + \gamma_R [2 - f_R(\Delta)]} \approx \frac{2e^2\gamma_L\gamma_R}{\gamma_L + \gamma_R} \frac{(V_L - V_R)/T}{2e^{\Delta/T} + 3 + e^{-\Delta/T}}. \end{aligned} \quad (5.32)$$

Note that the conductance peak is not symmetric. This is the consequence of correlations in transport of electrons with opposite spins through a single discrete state on the dot.